Nonlinear Control Oriented Boiler Modeling—A Benchmark Problem for Controller Design

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Abstract—This paper presents the development of a control oriented boiler model carried out on the basis of fundamental physical laws, previous efforts in boiler modeling, known physical constants, plant data, and heuristic adjustments. The resulting fairly accurate model is nonlinear, fourth order, and includes inverse response (shrink and swell effects), time delays, measurement noise models, and a load disturbance component. The model obtained can be directly used for the synthesis of model-based control algorithms as well as setting up a real-time simulator for testing of new boiler control systems and operator training.

I. INTRODUCTION

One of the most effective means of boiler efficiency enhancement is an improvement of the steam generation control system. An essential tool for such an improvement is a valid boiler model. Methods of obtaining such a model, however, are not readily found in an open literature and are often specific to a particular plant. Moreover, to the best of our knowledge there is no easily accessible presentation of a complete control-oriented boiler model containing all the manipulated and measured variables, disturbance models, uncertainties and constraints, with all the numerical values of the coefficients clearly given. The objective of this paper is to present a faithful control-oriented mathematical model for an industrial boiler that takes into account the coupling between the individual boiler subsystems with all the variables and constants explicitly defined. This model is currently used in a real time simulator of a steam generation system for the purposes of evaluation of various control algorithms as well as for operator training. The boiler model presented here displays all the essential features of the actual boiler dynamics, including nonlinearities, nonminimum phase behavior, instabilities, noise spectrum in the same frequency range as significant plant dynamics, time delays, and load disturbances. Because the boiler model faithfully representing a steam generation process must include such a vast array of the archetypal stumbling blocks in the controller design which are often left out of oversimplified academic problems while stubbornly surfacing in actual control engineering practice, the model presented here promises to be quite useful as a benchmark problem in the evaluation of various control algorithms. The researchers have used the model to design and evaluate a variety of control schemes; such as proportional, integral, and derivative (PID) control, GPC, and $H_{\infty}$ control in both real time application and in the off-line simulation.

The behavior of the model described in this paper accurately represents the significant dynamics of the boiler at Abbott Power Plant in Champaign, IL, in the normal regimes as well as many feasible abnormal ones. The model is control oriented in that all manipulated variables (inputs), all measured variables (outputs), all significant disturbances, and measurement noise are explicitly shown, model uncertainties are described, and constraints on the inputs and states are given. This paper, upon review of currently available models for steam generation, establishes the need for a control oriented model and presents a complete boiler model that predicts process response in terms of measurable outputs (drum pressure, drum water level, and excess oxygen in flue gas) to the major controllable inputs (air/fuel flow rates, feedwater flow rate) as well as the effect of disturbances (changed steam demand, sensor noise), model uncertainty (e.g., fuel calorific value variations, heat transfer coefficient variations, distributed dynamics of the steam generation), and constraints (actuator constraints, unidirectional flow rates, drum flooding). The paper is organized as follows. The development of the model is presented in Section II. The model enhancements, including the equation derivations and computation of the key model constants are described in Section III. Conclusions, the presentation of the final model, and simulation results are given in Section IV.

II. MODEL DEVELOPMENT

In developing the mathematical model presented in this paper we reviewed a number of analytical models which have been proposed for predicting the behavior of boilers. Recently, there has been growing activity in operator training and enhancement of boiler efficiency and performance through improved control systems. Although an essential component of these efforts is the development of an adequate mathematical model, the number of detailed, well-documented mathematical boiler models available in the current literature is rather limited. To the best of our knowledge, the models available in the open literature are essentially those presented by Bell and Åström [1]-[3]; McDonald et al. [8], Chawdry and Hogg [6], and Rubashkin and Khesin [11]. These models were compared and analyzed to determine their applicability to the problem of controller synthesis and real-time evaluation of controller performance for Abbott Power Plant.

These models can be categorized and compared by considering both the specific intended application of the model, and the developmental approach of the researchers. The equations
were developed on the basis of a combination of physical laws (first principles) such as mass and energy balances, heuristic knowledge of boiler behavior, and a data fit via numerical identification. The size and purpose of the boiler determines which subsystems of the physical processes require detailed modeling. The complexity of the models reviewed varies from simple linear low order ones to those of high order, coupled, and nonlinear. For example, in the modeling of the steam flow rate, the corresponding equations must be consistent with the structural configuration of the plant. This requirement is discussed in detail below. Also, an important consideration is to obtain a model that has a relatively low complexity while faithfully capturing the essential plant dynamics.

A. Units and Notation Used in the Boiler Modeling Literature

This paper uses SI units throughout. The English units are presented in parentheses. In the existing literature a variety of units is used. Whenever a reference is made to the previous work, the units used in this work are clearly indicated and converted to SI. The notation used in this paper is as follows. Constant parameters are represented by capital letters. The inputs, outputs, and states of the model are represented by boldface type. Intermediate variables are represented by lower case type.

B. Features of the Plant at Abbott

We consider boiler no. 2 at Abbott Power Plant in Champaign, IL, which is a dual fuel (oil and gas) fired unit used for both heating and electric power generation (cogeneration) and is rated at 22.096 kg/s (175,000 lb/hr) of steam at 2.24 MPa (325 PSI). The steam from the no. 2 boiler and the other gas boilers flows into a common header. A diagram of the inputs, outputs, and states of the model is shown in Fig. 1. An additional point of interest pertaining to the configuration of the Abbott boiler is the internal regulation of steam flow rate at the inlet of the turbine. From the operational standpoint, the output of boilers which generate industrial process steam is the steam flow rate, fluctuating in response to the internal pressure state and load demand determined by the users. When the steam passes to an electricity generating turbine, however, a governor valve regulates the steam flow rate to provide a steady electrical output. The most significant consequence of the latter configuration is reflected in the response of the boiler to a step input in fuel flow rate; namely, the steam flow rate generated by the boiler will exceed that taken out by the turbine and the difference will lead to a build up of pressure in the header in an integral fashion. In many cases, especially in the case of cogeneration plants such as Abbott, the actual situation reflects a combination of these two structurally distinct configurations.

C. The Boiler Control Problem

A properly functioning boiler must satisfy the following basic requirements: 1) a desired steam pressure must be maintained at the outlet of the drum (header pressure) despite variations in the quantity of steam demanded by users, 2) the water in the drum must be maintained at the desired level to prevent overheating of drum components or flooding of steam lines, and 3) the mixture of fuel and air in the combustion chamber must meet standards for safety, efficiency, and protection of the environment. This is accomplished by maintaining a desired percentage of oxygen in the stack in excess of that required for stoichiometric combustion, usually referred to as excess oxygen. Any model to be used for control system testing must contain at the very least these subsystems.

D. Summary of Available Models

McDonald et al. [8] developed a detailed nonlinear boiler model based on a “first principle” analysis and, therefore, the model parameters are readily related to physical parameters. These parameters were calculated using construction data, steam table data, and unit acceptance test data. The final model is a high order, coupled, complex, nonlinear model. For our purposes, this model uses information unavailable to us within the scope of the project and the setup at Abbott power plant, making the model almost impossible to fit to the plant. Chawdry and Hogg [6] developed a corresponding model through a two-stage recursive-least-squares algorithm with behavior closely matching that of the nonlinear equations of [8]. Although the results of [6] apply only to their particular system, the methods employed are helpful in setting up identification procedures for other systems. Rubashkin and Khesin took a different approach and considered the problem of modeling a boiler to effectively simulate regimes far outside.
of the normal operating range, such as the start up regime. Such models are particularly well suited for operator training and failure tolerance. While the approach of the paper is appealing, not enough technical information is provided to make direct use of the results and apply them to model Abbott Power plant.

In the study by Åström and Bell [2], several models developed by Morton and Price [10], Åström and Ecklund [4], and Bell and Åström [1] were reviewed and compared. Both simple linear models as well as high order and nonlinear ones were considered. In a later study, Åström and Bell developed a model [3] which relied more on first principles, and therefore, was likely to better represent the internal drum dynamics. Due to the relative complexity of the model and the general lack of developmental supporting literature, it is much more difficult to tailor the second model to the plant and perform the linearization and analysis that is required by the controller design. Therefore, the first of the two models by Åström and Bell is more appropriate as a foundation for our model.

III. MODEL ENHANCEMENT

On the basis of the structures and work described previously, the features of the model were developed to give a complete representation of the steam generation process at Abbott boiler no. 2. The nonlinear model was fit to match the dynamics of the Abbott boiler by modifying the boiler model developed by Åström and Bell [1]. To complete the model to suit our needs, the following changes and additions were made on the basis of physical laws, identification experiments, and heuristic knowledge of boiler dynamics. The decoupled turbine subsystem was removed from the equations presented in [1]. The following novel features were incorporated. A nonlinear combustion equation with a first-order lag was added to model the excess oxygen in the stack and the stoichiometric air-to-fuel ratio for complete combustion. Dynamic models of the plant measurement noise were obtained by subtracting the simulated outputs of the deterministic part of the equations from the actual output data using the same input data, and then fitting first-order linear models to match the frequency spectra of the residual. The load disturbance was modeled as a position of the imaginary valve at the output header as described in detail in Part C of this section.

To make these refinements of the boiler model and to enable the model to accurately describe the boiler dynamics at Abbott Power Plant, operating data was collected at Abbott for a pseudorandom binary sequence (PRBS) applied to the inputs of the system. This required the interface of the data collecting software with the equipment, data manipulation to obtain meaningful results from the raw data, and fitting the nonlinear equations for consistency with these results. The validity of the final model was verified through simulation and comparison with plant data.

A. Derivations of Nonlinear Equations

The equations below are a result of the several stages of correlating the simulated responses with actual plant data. The equations given below are coupled with those of Åström andBell [1] presented in Part B. The first group of equations relates the control input valve position to the input flow rates for the fuel ($q_f$), air ($q_a$), and feed water ($q_f w$) flow rates, respectively, (cf., Fig. 1)

$$q_f = QFCF u_1, \tag{1}$$

$$q_a = QACA u_2, \tag{2}$$

$$q_f w = QCFW u_3. \tag{3}$$

The differential equation for the drum pressure as presented in Bell and Åström [1] is given below

$$\dot{x}_1 = -CPI x_4 x_1^{CPI} + CIP u_1 - CP3 u_3 + CPA \tag{4}$$

and depends on the exogenous variable $x_4$, (referred to as control valve position in [4]), fuel flow rate $u_1$, and water flow rate $u_3$.

The oxygen level equation given below was developed by first theoretically determining the steady-state oxygen level for a given air flow rate and fuel flow rate. Assuming complete combustion, the percent oxygen remaining after combustion, denoted as $O_2$, can be expressed as

$$O_2 = \frac{100(q_a - q_f \text{FAR})}{(q_a + q_f) \text{AIRO2}} \tag{5}$$

where FAR is the air to fuel mass ratio for complete combustion, and AIRO2 is the mass ratio of air to oxygen in atmospheric air. By assuming a first-order lag with time constant $TAIR$, the continuous time differential equation of the oxygen level state, can be expressed as

$$\dot{x}_2 = \frac{100(q_a - q_f \text{FAR})}{(q_a + q_f) \text{AIRO2} - x_2} \frac{1}{TAIR}. \tag{6}$$

Upon observing the performance of the nonlinear oxygen level equation, it became obvious that the simple equation initially developed could not capture with sufficient accuracy the nonlinear behavior of the plant at the several operating points represented by the response data to step and pseudorandom inputs. The constant, $FAR$, when calculated to match several local steady state operating levels, showed wide variations, but it was noticed that these variations were directly correlated with the oxygen level operating point. To capture this phenomenon, a function far (24) which made the steady state oxygen level of (5) match the oxygen level data for a variety of operating points was calculated and plotted versus the oxygen level. This plot, shown in Fig. 2, exhibits a nearly linear relationship, described by (24).
In determining the internal relationships, the steam flow rate is a given quantity, and treated as a disturbance input, but in considering simulations for control system testing, we would like to compute the steam flow rate for a given set of control inputs. Since a measurement of the load is physically nonexistent, a model of the load disturbance was developed. Using the steam flow rate (7) of Åström and Ecklund [4] which relates the header pressure and an imaginary valve position to the header steam flow rate, the imaginary valve position can be calculated for measured values of pressure and steam flow rate. Although conceptually it is attractive to consider the load level to be completely independent of the fuel flow rate, and consequently, independent of the steam flow rate generated by the boiler, in reality, the net “user” is not “strong enough” to enforce such an independence, and if the boiler provides more steam than necessary the user will be forced to accept part of the steam excess. This means that the load level, \( x_4 \), must be correlated with the fuel flow rate in the actual operation. Such a correlation was observed in the actual plant data (cf., Fig. 6). To capture this correlation, a first-order dynamic model, (28), was formulated for calculating the imaginary valve position as a function of the fuel flow rate plus a colored noise term. The steam flow rate, denoted as \( q_s \), as a function of pressure and the exogenous variable \( x_4 \) is

\[
q_s = (x_4 CQS1 - CQS2)x_1. \tag{7}
\]

The load parameter \( x_4 \) was computed from (7) using the plant data for steam flow rate and pressure, and then the dependence on fuel flow rate \( u_1 \) was estimated from the plant data, yielding the steady-state relation

\[
x_4 = CD11 u_1 + CD12. \tag{8}
\]

This relation also contains a lag, with time constant identified as \( TD1 \)

\[
x_4 = -(x_4 - CD11 u_1 - CD12) \frac{1}{TD1}. \tag{9}
\]

### B. Equations Developed in the Previous Work by Åström and Bell [2]

Equations (1)–(9) presented above are combined with (10)–(15) briefly described below. These are described in detail in Åström and Bell [2]

\[
rhs = CS1 x_1 + CS2 \tag{10}
\]

\[
msd = \frac{KB e f - R q f w + q_s K}{1 + K} \tag{11}
\]

\[
vwd = VW VT x_3 + CVWD1 a_1 + SD msd \tag{12}
\]

\[
a_1 = \frac{1}{1 - VW} \tag{13}
\]

\[
e f = CU11 q f + CU12 \tag{14}
\]

\[
x_3 = QCFW u_3 - q_s \tag{15}
\]

The final group of equations are the output equations, which provide the proper scaling to match the Abbott system outputs

\[
y_1 = SCP x_1, \text{ describes the pressure (PSI)}, \tag{16}
\]

\[
y_2 = x_3, \text{ describes the oxygen level (percent)}, \tag{17}
\]

\[
y_3 = SCW CXW1 (vwd - CXW2), \text{ describes the water level, (in)} \tag{18}
\]

\[
y_4 = q_s \text{ is the steam flow rate (Kg/s) from (7)}. \tag{19}
\]

### C. Techniques Used in Parameter Determination

The final simulation model included a number of constants, some having clear physical interpretation, and others obtained via a curve fit to experimental data or steam table data. As a first step, as much information as possible was obtained from known data. This included the parameters which can be computed from construction data, steam table data, and known quantities. Values obtained from steam tables were computed at the nominal operating point conditions. We attempted to isolate the parameters that had to be determined through identification, and to access them through the identified transfer functions. Using a symbolic linearization and continuous time transfer functions, these parameters were rendered identifiable. Several steps and mappings were involved in obtaining the discrete transfer functions in terms of the parameters of the full nonlinear equations. Ideally we intended to reduce the nonlinear equations to a linear transfer function form where each unknown parameter block corresponded to a constant numerical value in the identified transfer functions, each constant constant in the identified transfer functions then produced one algebraic equation, which was solvable by inserting all the known quantities into the equations. In the case of an over-specified constant, such as the steam flow rate constant, it was possible to identify parameters that appeared exclusively as a group in both the nonlinear and linearized equations. For the under-specified constants, a heuristic adjustment was necessary in the simulation phase of the model development. Testing the resulting model with an off-line simulator provided plots exhibiting the behavior of the model at this stage of the development. For example, a consistent bias of the simulated oxygen level from the combustion equation and the underactivity of the output in the water level equation were the motivation for further refinements through heuristic model adjustment.

### D. Computation of Constants

The constants in the nonlinear set of equations were computed in several ways: 1) through determination of known physical values, 2) through identification of grouped coefficients, and 3) through heuristic adjustments. The availability of the physical constants, and the effectiveness of the individual identification procedures dictated which method would be
used for each equation. The information obtained from the initial step responses included major time constants, time delays, and the general attributes of the system response. The PRBS data provided a much richer input signal used to obtain a more consistent model through identification. The identification based on the data provided the transfer functions which were used in fitting the nonlinear model. Below is a description of the computations unique to our model derivation.

The pressure equation identification model was chosen to be first order and dependent on the fuel, the water, and the steam flow rates. This provided a very close correlation with the actual plant data. Because of the satisfactory data fit, the transfer function coefficients were directly used to fit the nonlinear model by equating the identified coefficients with those of the linearized plant.

The constants labeled CP* are the constants in the pressure nonlinear equation. In the symbolic linearization of the equations, the constants appear explicitly in the transfer functions. To obtain the parameter values, we equated the transfer function coefficients from the symbolic linearization (on the left) with the identified transfer functions (on the right). For example

\[
\frac{Y_1(s)}{U_1(s)} = \frac{C_{11}B_{11}}{s - A_{11}}
\]

\[
\frac{Y_1(s)}{U_3(s)} = \frac{C_{11}B_{13}}{s - A_{11}}
\]

\[
\frac{Y_1(s)}{Q_s(s)} = \frac{C_{11}B_{14}}{s - A_{11}}
\]

After substituting all the previously determined constants, the equations were solved for the remaining variables

\[
CP_1 = 0.0280, \quad CP_3 = 0.01348
\]

and

\[
\frac{CP_1}{CQS_1} = 0.00558.
\]

The nonlinearities of the oxygen level equation caused some problems for the identification procedure. The transfer functions obtained for the step response data and the PRBS data were very different, making the results too particular to generalize to the nonlinear equations. The heuristic adjustments to the nonlinear equations gave very good steady-state performance, so only the lag of the equation was used according to the identification results. The numerator dynamics were computed via the theoretical computation of the excess oxygen to determine the form of the equation and a steady state analysis of the plant data to compute the actual coefficients.

**TAIR** is the time lag for the combustion dynamics. By equating the denominator of the symbolic linearization with the identified transfer function denominator, **TAIR** was computed as

\[
\frac{Y_2(s)}{U_1(s)} = \frac{C_{22}B_{21}}{s - A_{22}}
\]

\[
= \frac{-5.88}{s + 0.1540}
\]

therefore

\[-A_{22} = \frac{1}{TAIR} = 0.1540\]

and

\[TAIR = 6.492 \text{s.}\]

The air to fuel ratio (**FAR**) was the only uncertain parameter in the oxygen level equation development. As a result, the nonlinear equation in steady state had to be solved using the plant inputs and outputs to obtain the actual **FAR** at several equilibrium points. This showed that the **FAR** in the model varied linearly with the oxygen level as shown in Fig. 2, therefore, a function was fit to these data to compute the **FAR** for complete combustion as a function of oxygen level.

This fit is given by

\[\text{FAR} = 0.310629O_2 + 16.2983\]

where

\[F_{A1} = 0.310629\]

and

\[F_{A2} = 16.2983.\]

Identification for the water level equation was somewhat less successful. The inputs of water flow rate and steam flow rate affected the level, but the first-order fit showed a poor correlation with the actual data. A third-order fit showed closer short term behavior, but even with the higher order, the fit was not acceptable. The nonlinear equation which was computed from first principles, provided a better fit to the actual plant data, and the coefficients were calculated from construction specifications for the boiler, and steam table constants.

Since the two constants **CP1**/**CQS1** were clustered together wherever they appeared, identification could not determine them both uniquely. A reasonable way to choose **CQS1** is to select it to yield a reasonable valve position, **x4**, for the steam flow rate and pressure data used in (7). Assuming that the normal load varies as \(0 < x_4 < 1\), we obtain

\[CQS_1 = 0.85663\]

and

\[CQS_2 = 0.18128\]
which yields the value of $x_4$ within reasonable bounds. Using this \( CQ_{S1} \), we can compute

\[
CP_1 = 0.00558
\]

and

\[
CQ_{S1} = 0.00478.
\]

The lag coefficient for the load level differential equation is \( TD_1 \). A first order linear transfer function was identified from the relationship between fuel flow rate and load level data, yielding

\[
TD_1 = 25.0\text{s}.
\]

The constant \( CP_4 \) accounts for subtle discrepancies in the operating point of the nonlinear pressure equation. This constant drops out of the transfer function in the linearization, so it does not affect the local behavior. \( CP_4 \) was chosen to minimize the long term average offset between the simulated drum pressure and the actual recorded data

\[
CP_4 = 0.02493.
\]

The increase in water volume for a unit increase in evaporation rate is \( SD \). This parameter was heuristically adjusted to give a reasonable swell (the initial, temporary rise of water level) for a step in fuel flow rate or steam flow rate

\[
SD = 0.159.
\]

The constants which relate the fuel flow rate to the energy flow rate to the system are \( CU_{11} \) and \( CU_{12} \). To calculate these quantities, the net energy change from the feedwater entering the system to the steam leaving the system was calculated from several steady state operating data points. The best fit to these data is the line

\[
e_f = 37633.0 q_f + 174.
\]

Therefore

\[
CU_{11} = 37633.0
\]

and

\[
CU_{12} = 174.
\]

E. Noise Models

The measured outputs at Abbot Power Plant contain measurement noise resulting from high frequency variations in signals levels (e.g., boiling effect on water surface, pressure waves), or corruption of sensor signals (e.g., pneumatic line leaks or vibrations, electrical line noise). To reproduce this effect, dynamic models of the plant measurement noise were added to the outputs of (25)–(32). If the simulated deterministic outputs of the equations are subtracted from the actual output data for a simulation using identical input data, the residual which represents the part of the output that is unaccounted for in the simulation can be isolated. The noise values added to \( x_4 \) in (32) were computed from the residual of this procedure in (9). The noise models of Table I. are computed by fitting first-order linear models to match the frequency spectra of this residual.

### Table I

<table>
<thead>
<tr>
<th>Equation Coefficients</th>
<th>( c_{11} )</th>
<th>( c_{12} )</th>
<th>( c_{13} )</th>
<th>( c_{14} )</th>
<th>( c_{76} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{11} )</td>
<td>-0.00478</td>
<td>-0.85663</td>
<td>0.0251950</td>
<td>0.0011850</td>
<td>0.15479075</td>
</tr>
<tr>
<td>( C_{12} )</td>
<td>0.945853</td>
<td>0.029988</td>
<td>0.04004</td>
<td>0.02998</td>
<td>-0.1048569</td>
</tr>
<tr>
<td>( C_{13} )</td>
<td>0.01348</td>
<td>0.0180888</td>
<td>-0.00478</td>
<td>0.005331776</td>
<td>1.272063</td>
</tr>
<tr>
<td>( C_{14} )</td>
<td>0.02493</td>
<td>0.945853</td>
<td>0.02432</td>
<td>-0.154385</td>
<td>-0.00478</td>
</tr>
<tr>
<td>( C_{76} )</td>
<td>0.280</td>
<td>0.02432</td>
<td>0.029988</td>
<td>-0.154385</td>
<td>-0.00478</td>
</tr>
</tbody>
</table>

F. Uncertainty in Plant Dynamics

The model displays coupling effects such as a strong influence of fuel flow rate on all four outputs, the drifting of process parameters because of corrosion and wear, transportation delays in the piping and stack resulting in varying dead time, changing fuel calorific value, nonminimum phase behavior, and instability in drum dynamics, sensor noise with frequency range overlapping that of significant plant dynamics, system nonlinearities, and saturation nonlinearities produced by actuator constraints. Also, the changes in the unmeasurable load disturbance, referred to as user steam demand, have a significant effect on the operation of the system. In addition, the setpoints for the boiler change several times during every 24 h period to maintain the optimal operating regimes. Thus, the boiler control problem consists of tracking and regulation under the complicating conditions described above. This set of requirements in the control problem, along with the plant characteristics makes the model a useful tool for testing experimental control schemes on a realistic problem.

IV. Conclusions

Identification is the most effective way to deal with the differential equation describing the pressure dynamics, because, although the transfer functions constitute a simple plant approximation, their coefficients are comprised of a complex combination of physical parameters difficult to calculate. The water level equation, on the other hand is defined by complex processes made up of parameters with directly accessible physical interpretations. This makes this subsystem very difficult to model via pure identification, as was found during our identification experiments which gave quite inconsistent results for the water level subsystem. This also explains why Åström and Bell chose the nonlinear model to have the pressure equation defined by an identified data fit and the water level based on first principles. In Åström and Bell [1], the variable \( x_4 \) is labeled as the load disturbance \( d_1 \), but the only related measurable quantity at Abbott Power Plant is the steam flow rate, which is related to \( x_4 \) by (32). The values of \( x_4 \) were calculated from (32) using the plant measurements of pressure, \( x_1 \), and steam flow rate, \( y_4 \). The first order model (28) for the auxiliary variable, \( x_4 \), was then constructed including a dependence on the fuel flow rate, \( u_1 \).
plus a random colored noise term, \( n_5 \). The time delays, \( \tau_i \), of the system include measurement instrumentation delays, process delays, and transport delays, that result in a complex nonlinear relation for \( \tau_i \) which depends on travel distances, gas flow rates, flow resistance, and gas buoyancy relationships. Because we observed only slowly changing slight variations in the plant data delays, we consider constant delays in the controller design and simulations.

The explicit model equations have the form

\[
\begin{align*}
\dot{x}_1(t) &= c_{11}x_4(t)x_1^{3/4}(t) + c_{12}u_1(t - \tau_1) \\
\dot{x}_2(t) &= c_{21}x_2(t) + [c_{22}u_2(t - \tau_2) - c_{23}u_1(t - \tau_1)] \\
\dot{x}_3(t) &= c_{31}x_1(t) - c_{32}u_1(t - \tau_1) \frac{x_2(t)}{c_{33}u_2(t - \tau_2)} \\
\dot{x}_4(t) &= c_{41}x_4(t) + c_{42}u_1(t - \tau_1) + c_{43} + n_5(t),
\end{align*}
\]

where

\[
\begin{align*}
x_1 &:= \text{drum pressure state (kgf/cm}^2\text{); } \\
x_2 &:= \text{measured drum pressure (PSI);} \\
x_3 &:= \text{excess oxygen level and its state, respectively, (percent);} \\
x_4 &:= \text{system fluid density (kg/m}^3\text{); } \\
x_5 &:= \text{drum water level (in);} \\
x_6 &:= \text{steam flow rate (kg/s);} \\
x_7 &:= \text{fuel, air, and feed water flow rate inputs which take values between 0-1;} \\
x_8 &:= \text{the exogenous variable related to the load disturbance intensity (0-1);} \\
\end{align*}
\]

The coefficients in (25)-(32) are explicitly listed below. They represent a combination of plant specifications, construction data, steam table data, and physical parameters. To further improve and validate the model, additional identification tests spanning several operating points should be performed on the boiler. These tests would refine the performance of the nonlinear model in various regimes. The nonlinear model fit to these various linear models would be much more complex, and would require other techniques such as nonlinear regression to identify the individual parameters identified as lumped groups.

The noise models are represented by

\[
\begin{align*}
n_1 &= 0.075s + 0.1 \\
n_2 &= 0.019s + 0.001 \\
n_3 &= 0.105s + 0.038 \\
n_4 &= 0.01s + 0.0001 \\
n_5 &= 0.0003s + 0.003 \\
wd &= s + 0.0075 \\
\end{align*}
\]

where \( n_i, i = 1, \ldots, 5 \) is the colored noise, \( w_i \) is unit variance white noise, and \( wd \) is the deterministic part of the disturbance that defines the load level.

The operating point around which the plant is linearized is

\[
\begin{align*}
x^0 &= [22.5 2.5 621.17 0.6941]^T, \\
y^0 &= [320 2.5 0.0 9.3053]^T, \\
u^0 &= [0.32270 0.39503 0.37404]^T.
\end{align*}
\]

State-space plant linearization is given by

\[
\dot{x} = A_p x + B_p u, \\
y = C_p x + D_p u
\]

where

\[
\begin{align*}
A_p &= \begin{bmatrix}
-0.00509 & 0 & 0 & -0.1588 \\
0 & -2.2062 & 0 & 0 \\
-0.01216 & 0 & 0 & -0.5672 \\
0 & 0 & 0 & -0.040
\end{bmatrix}, \\
B_p &= \begin{bmatrix}
0.2800 & 0 & -0.01348 & 0 \\
-9.375 & 7.658 & 0 & 0 \\
0 & 0 & 0.7317 & 0 \\
0.02999 & 0 & 0 & 0.040
\end{bmatrix}, \\
C_p &= \begin{bmatrix}
14.21 & 0 & 0 & 0 \\
0 & 1.0 & 0 & 0 \\
0.3221 & 0 & 0.1433 & 11.16 \\
0.4133 & 0 & 0 & 19.28
\end{bmatrix}, \\
D_p &= \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1.272 & 0 & -0.2080 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
The model with the adjusted parameters was tested via off-line simulations using the C code simulator. The plots of the data and the final nonlinear model fit are shown in the figures. In Fig. 3, the drum pressure, and in Fig. 4, the oxygen level show good accuracy, both in short term and long term characteristics. The drum water level comparison of Fig. 5 shows better long term matching than the short term behavior. In each of these plots, the simulated plant is based on the measured inputs without the addition of noise models. The plots representing the actual data, therefore, shows more pronounced noise in the signals. The last plot, Fig. 6, shows the relative activity in the PRBS inputs, which were actuated during the testing, and then later used for the simulation. The plot includes the randomly actuated fuel flow rate and water flow rate, the constant air flow rate, and the steam flow rate which reacted in response to the load level of the plant.

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REFERENCES